Reg. No. :

# Question Paper Code : 91783

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Third Semester

Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to All Branches)

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Form the partial differential equation by eliminating the arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2 + 1$.

2. Find the complete integral of $p + q = x + y$.

3. State the Dirichlet's conditions for a function f(x) to be expanded as a Fourier series.

4. Expand $f(x) = 1$, in $(0, \pi)$ as a half-range sine series.

5. Write all possible solutions of one dimensional heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$.

6. Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ where $u(x, 0) = 6e^{-3x}$.

7. If the Fourier transform of f(x) is $\Im(f(x)) = F(s)$, then show that $\Im(f(x - a)) = e^{ias} F(s)$.

8. Find the Fourier sine transform of 1/x.

9. Find the Z – transform of $\frac{1}{n+1}$

10. State the final value theorem of Z transforms.

PART – B (5×16=80 Marks)

11. a) i) Find the general solution of $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. (8)

ii) Find the general solution of $(D^2 + 2DD' + D'^2) z = x^2 y + e^{x-y}$. (8)

(OR)

b) i) Find the general solution of $z = px + qy + p^2 + pq + q^2$. (8)

ii) Find the general solution of $(D^2 - 3DD' + 2D'^2 + 2D - 2D') z = \sin (2x + y)$. (8)

12. a) i) Find the Fourier series expansion of the following periodic function $f(x) = \begin{cases} 2 + x & -2 \le x \le 0 \\ 2 - x & 0 < x \le 2 \end{cases}$. of period 4 Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$. (8)

ii) Find the complex form of Fourier series of $f(x) = e^{ax}$ in the interval $(-\pi, \pi)$ where a is a real constant. Hence, deduce that $\sum_{n=-\infty}^{\infty} \frac{(-1)^n}{a^2 + n^2} = \frac{\pi}{a \sinh a\pi}$. (8)

(OR)

b) i) Find the half range cosine series of $f(x) = (\pi - x)^2$, $0 < x < \pi$. Hence find the sum of the series $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \ldots$ (8)

ii) Determine the first two harmonics of Fourier series for the following data. (8)

| | | | | | | |
|---|---|---|---|---|---|---|
| x : | 0 | $\frac{\pi}{3}$ | $\frac{2\pi}{3}$ | $\pi$ | $\frac{4\pi}{3}$ | $\frac{5\pi}{3}$ |
| f(x) : | 1.98 | 1.30 | 1.05 | 1.30 | –0.88 | –0.25 |

13. a) A tightly stretched string of length '$l$' with fixed end points is initially at rest in its equilibrium position. If it is set vibrating by giving each point a velocity $y_t (x, 0) = \upsilon_0 \sin\left(\frac{3\pi x}{l}\right) \cos\left(\frac{\pi x}{l}\right)$, where $0 < x < l$. Find the displacement of the string at a point, at a distance x from one end at any instant 't'. (16)

(OR)

b) A square plate is bounded by the lines x = 0, x = 20, y = 0, y = 20. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 20) = x (20 - x)$, $0 < x < 20$, while the other three edges are kept at 0°C. Find the steady state temperature distribution u(x, y) in the plate. (16)

14. a) i) Find the Fourier Transform of f(x) if $f(x)=\begin{cases}1-|x|, & |x|<1\\ 0, & |x|>1\end{cases}$ and hence evaluate the integral $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt$. **(10)**

ii) State and prove convolution theorem for Fourier transforms. **(6)**

(OR)

b) i) Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using transforms. **(6)**

ii) Find the Fourier cosine transform of $f(x) = e^{-a^2x^2}$ and hence find $F_S\left[xe^{-a^2x^2}\right]$. **(10)**

15. a) i) Find $Z(r^n \cos n\theta)$ and $Z^{-1}\left[\left(1-az^{-1}\right)^{-2}\right]$. **(8)**

ii) Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-1/2)(z-1/4)}\right]$. **(8)**

(OR)

b) i) Using Z-transform, solve the difference equation $x(n+2) - 3x(n+1) + 2x(n) = 0$ given that $x(0) = 0$, $x(1) = 1$. **(8)**

ii) Using residue method, find $Z^{-1}\left[\frac{z}{z^2-2z+2}\right]$. **(8)**

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